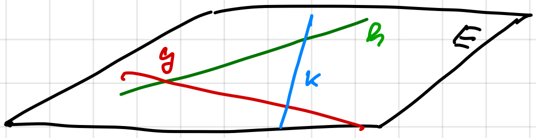
Abstandberechnung mit Hilfe von Pyramiden

Geg.: $P(-4|8|-2)$

$$g: \vec{x} = \begin{pmatrix} 11 \\ 6 \\ -6 \end{pmatrix} + r \cdot \begin{pmatrix} 8 \\ 2 \\ -7 \end{pmatrix}$$

$$h: \vec{x} = \begin{pmatrix} 1 \\ -4 \\ -1 \end{pmatrix} + s \cdot \begin{pmatrix} 1 \\ 4 \\ 1 \end{pmatrix}$$

$$k: \vec{x} = \begin{pmatrix} -11 \\ 8 \\ 17 \end{pmatrix} + t \cdot \begin{pmatrix} -2 \\ 2 \\ 3 \end{pmatrix}$$



a) Drei Geraden liegen genau dann in einer gemeinsamen Ebene, wenn sie sich paarweise schneiden!

$g \cap h$:

$$\begin{aligned} \text{I}: &\; 11 + 8r = 1 + s \\ \text{II}: &\; 6 + 2r = -4 + 4s \\ \text{III}: &\; -6 - 7r = -1 + s \end{aligned}$$

I − III: $17 + 15r = 2 \Rightarrow r = -1$

in II: $6 - 2 = -4 + 4s \Rightarrow s = 2$

$g \cap k$:

$$\begin{aligned} \text{I}: &\; 11 + 8r = -11 - 2t \\ \text{II}: &\; 6 + 2r = 8 + 2t \\ \text{III}: &\; -6 - 7r = 17 + 3t \end{aligned}$$

I + II: $17 + 10r = -3 \Rightarrow r = -2$

in III: $-6 + 14 = 17 + 3t \Rightarrow t = -3$

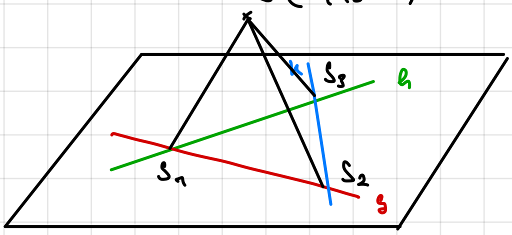
$h \cap k$:

$$\begin{aligned} \text{I}: &\; 1 + s = -11 - 2t \\ \text{II}: &\; -4 + 4s = 8 + 2t \\ \text{III}: &\; -1 + s = 17 + 3t \end{aligned}$$

I + II: $-3 + 5s = -3 \Rightarrow s = 0$

in III: $-1 = 17 + 3t \Rightarrow t = -6$

$\Rightarrow$ Die Geraden liegen in einer Ebene!



$S_{gh}(3|4|1) = S_1$

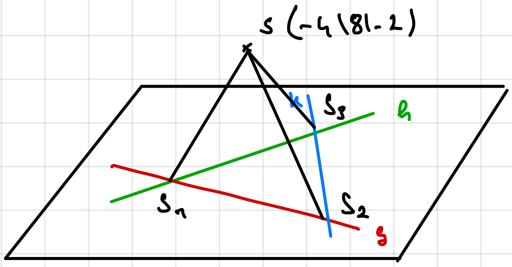
$S_{gk}(-5|2|8) = S_2$

$S_{hk}(1|-4|-1) = S_3$

$$\vec{S_1S_2} = \begin{pmatrix} -8 \\ -2 \\ 7 \end{pmatrix}$$

$$\vec{S_1S_3} = \begin{pmatrix} -2 \\ -8 \\ -2 \end{pmatrix}$$

$$\vec{S_1S} = \begin{pmatrix} -7 \\ 4 \\ -3 \end{pmatrix}$$



$S_{gh}(3|4|1) = S_1$

$S_{gk}(-5|2|8) = S_2$

$S_{hk}(1|-4|-1) = S_3$

$$\overrightarrow{S_1S_2} = \begin{pmatrix}-8\\-2\\7\end{pmatrix} \quad \overrightarrow{S_1S_3} = \begin{pmatrix}-2\\-8\\-2\end{pmatrix} \quad \overrightarrow{S_1S} = \begin{pmatrix}-7\\4\\-3\end{pmatrix}$$

$$V_{Pyr} = \frac{1}{6} \cdot \left| \left(\overrightarrow{S_1S_2} \times \overrightarrow{S_1S_3}\right) \circ \overrightarrow{S_1S} \right| = \frac{1}{6} \cdot \left| \left[\begin{pmatrix}-8\\-2\\7\end{pmatrix} \times \begin{pmatrix}-2\\-8\\-2\end{pmatrix} \right] \circ \begin{pmatrix}-7\\4\\-3\end{pmatrix} \right|$$

$$= \frac{1}{6} \left| \left[\begin{pmatrix}4+56\\-14-16\\64-4\end{pmatrix} \circ \begin{pmatrix}-7\\4\\-3\end{pmatrix} \right] \right| = \frac{1}{6} \left| \left[\begin{pmatrix}60\\-30\\60\end{pmatrix} \circ \begin{pmatrix}-7\\4\\3\end{pmatrix} \right] \right| =$$

$$= \frac{1}{6} \left| -420 - 120 + 180 \right| = \underline{\underline{60 \text{ VE}}}$$

$$V_{Pyr} = \frac{1}{3} G \cdot h = \frac{1}{3} \cdot \frac{1}{2} \cdot \left| \overrightarrow{S_1S_2} \times \overrightarrow{S_1S_3} \right| \cdot h = \frac{1}{6} \cdot \left| \begin{pmatrix}60\\-30\\60\end{pmatrix} \right| \cdot h$$

$$= \frac{1}{6} \cdot 90 \cdot h \overset{!}{=} 60 \text{ VE}$$

$$\Rightarrow \underline{\underline{h = 4}} \text{ LE}$$